Skills Practice

Riding a Ferris Wheel
Introduction to Circles

Vocabulary
Identify an instance of each term in the diagram.

1. circle

2. center of the circle

3. chord of the circle

4. diameter

5. secant of the circle

6. tangent of the circle

7. point of tangency

8. central angle

9. inscribed angle

10. arc

11. major arc
12. minor arc

Problem Set

Identify the indicated part of each circle. Explain your answer.

1. \( \overline{OA} \)

\( \overline{OA} \) is a radius. It is a line segment that connects the center \( O \) to a point on the circle, \( A \).

2. \( \overline{GE} \)

3. \( O \)

4. \( \overline{NP} \)

5. \( \overline{AB} \)

6. \( D \)
Use circle \( P \) to answer each question.

11. Which point(s) are located in the interior of the circle?
   Point \( P \) is in the interior of circle \( P \).

12. Which point(s) are located on the circle?

13. Which point(s) are located in the exterior of the circle?

14. Why isn't \( MR \) a diameter?

15. How are \( MR \) and \( AR \) alike?
16. How are $\overline{MR}$ and $\overline{AR}$ different?

Identify each angle as an inscribed angle or a central angle.

17. $\angle URE$
   
   Angle $URE$ is an inscribed angle.

18. $\angle ZOM$

19. $\angle KOM$

20. $\angle ZKU$

21. $\angle MOU$

22. $\angle ROK$

Classify each arc as a major arc, a minor arc, or a semicircle.

23. $\widehat{AC}$
   
   $\widehat{AC}$ is a minor arc.

24. $\widehat{DE}$

25. $\widehat{FHI}$

26. $\widehat{JML}$
27. \( NPQ \)

28. \( TRS \)

Draw the part of a circle that is described.

29. Draw chord \( \overline{AB} \).

30. Draw radius \( \overline{OE} \).

31. Draw secant \( \overline{GH} \).

32. Draw a tangent at point \( J \).

33. Label the point of tangency \( A \).

34. Label center \( C \).
35. Draw inscribed angle $\angle FDG$.

36. Draw central angle $\angle HOI$.

37. Draw major arc $\widehat{KNM}$.

38. Draw minor arc $\widehat{RQ}$.
Skills Practice

Name _____________________________________________ Date ____________________

Holding the Wheel
Central Angles, Inscribed Angles, and Intercepted Arcs

Vocabulary
Define each term in your own words.

1. degree measure of a minor arc

2. adjacent arcs

3. Arc Addition Postulate

4. intercepted arc

5. Inscribed Angle Theorem

6. Parallel Lines-Congruent Arcs Theorem
Problem Set

Determine the measure of each minor arc.

1. \( \widehat{AB} \)

The measure of \( \widehat{AB} \) is 90°.

2. \( \widehat{CD} \)

3. \( \widehat{EF} \)

4. \( \widehat{GH} \)

5. \( \widehat{IJ} \)

6. \( \widehat{KL} \)
Determine the measure of each central angle.

7. \( \angle XYZ \)

\[ m\angle XYZ = 80^\circ \]

8. \( \angle BGT \)

9. \( \angle LKJ \)

10. \( \angle FMR \)

11. \( \angle KWS \)

12. \( \angle VIQ \)
Determine the measure of each inscribed angle.

13. \( m\angle XYZ \)

![Diagram of \( m\angle XYZ \)](image)

\( m\angle XYZ = 75^\circ \)

14. \( m\angle MTU \)

![Diagram of \( m\angle MTU \)](image)

15. \( m\angle KLS \)

![Diagram of \( m\angle KLS \)](image)

16. \( m\angle DVA \)

![Diagram of \( m\angle DVA \)](image)

17. \( m\angle QBR \)

![Diagram of \( m\angle QBR \)](image)

18. \( m\angle SGI \)

![Diagram of \( m\angle SGI \)](image)
Determine the measure of each intercepted arc.

19. $m\widehat{KM}$

20. $m\widehat{LU}$

\[ m\widehat{KM} = 108^\circ \]

21. $m\widehat{QW}$

22. $m\widehat{TV}$

23. $m\widehat{ME}$

24. $m\widehat{DS}$
Calculate the measure of each angle.

25. The measure of $\angle AOB$ is $62^\circ$. What is the measure of $\angle ACB$?

$$m \angle ACB = \frac{1}{2} (m \angle AOB) = \frac{62^\circ}{2} = 31^\circ$$

26. The measure of $\angle COD$ is $98^\circ$. What is the measure of $\angle CED$?

27. The measure of $\angle EOG$ is $128^\circ$. What is the measure of $\angle EFG$?
28. The measure of $\angle GOH$ is $74^\circ$. What is the measure of $\angleGIH$?

29. The measure of $\angle JOK$ is $168^\circ$. What is the measure of $\angle JIK$?

30. The measure of $\angle KOL$ is $148^\circ$. What is the measure of $\angle KML$?
Use the given information to answer each question.

31. In circle C, \( m\widehat{XZ} = 86^\circ \). What is \( m\widehat{WY} \)?

32. In circle C, \( m\angle WCX = 102^\circ \). What is \( m\widehat{YZ} \)?

33. In circle C, \( m\widehat{WZ} = 65^\circ \) and \( m\widehat{XZ} = 38^\circ \). What is \( m\angle WCX \)?
34. In circle $C$, $m\angle WCX = 105^\circ$. What is $m\angle WYX$?

35. In circle $C$, $m\angle WCY = 83^\circ$. What is $m\angle XCZ$?

36. In circle $C$, $m\angle WYX = 50^\circ$ and $m\angle XYZ = 30^\circ$. What is $m\overline{WXZ}$?
Skills Practice

Name _____________________________________________  Date ____________________

Manhole Covers
Measuring Angles Inside and Outside of Circles

Vocabulary
Identify the similarities and differences between the terms.

1. Interior Angles of a Circle Theorem and Exterior Angles of a Circle Theorem

Problem Set
List the two intercepted arcs for the given angle.

1. \( \angle AEB \)
2. \( \angle MRN \)
3. \( \angle VYU \)
4. \( \angle KNL \)
Write an expression for the measure of the given angle.

7. \( m\angle RPM \)

\[
m\angle RPM = \frac{1}{2}(m\widehat{RM} + m\widehat{QN})
\]

9. \( m\angle JNK \)

10. \( m\angle UWV \)
11. \( m\angle SWT \)

\[ \text{Diagram of circle with points } S, W, T, U, V, O \]

12. \( m\angle HJI \)

\[ \text{Diagram of circle with points } F, G, O, J, I, H \]

List the intercepted arc(s) for the given angle.

13. \( \angle QMR \)

\[ \text{Diagram of circle with points } M, N, O, P, Q, R \], \( \overarc{NP} \), \( \overarc{QR} \)

14. \( \angle RSU \)

\[ \text{Diagram of circle with points } R, S, T, U, O, P \]

15. \( \angle FEH \)

\[ \text{Diagram of circle with points } E, F, G, H, I, O \]

16. \( \angle ZWA \)

\[ \text{Diagram of circle with points } X, Y, Z, A, W, O \]
17. \( \angle BDE \)

18. \( \angle JLM \)

Write an expression for the measure of the given angle.

19. \( m\angle DAC \)

\[
m\angle DAC = \frac{1}{2}(m\widehat{DEC} - m\widehat{BC})
\]

20. \( m\angle UXY \)

21. \( m\angle SRT \)

22. \( m\angle FJG \)
Create a two-column proof to prove each statement.

25. Given: Chords $\overline{AE}$ and $\overline{BD}$ intersect at point $C$.

Prove: $m\angle ACB = \frac{1}{2} (m\widehat{AB} + m\widehat{DE})$

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
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<tbody>
<tr>
<td>1. Chords $\overline{AE}$ and $\overline{BD}$ intersect at point C.</td>
<td>1. Given</td>
</tr>
<tr>
<td>2. Draw chord $\overline{AD}$</td>
<td>2. Construction</td>
</tr>
<tr>
<td>3. $m\angle ACB = m\angle D + m\angle A$</td>
<td>3. Exterior Angle Theorem</td>
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<tr>
<td>4. $m\angle A = \frac{1}{2} m\widehat{DE}$</td>
<td>4. Inscribed Angle Theorem</td>
</tr>
<tr>
<td>5. $m\angle D = \frac{1}{2} m\widehat{AB}$</td>
<td>5. Inscribed Angle Theorem</td>
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<tr>
<td>6. $m\angle ACB = \frac{1}{2} m\widehat{DE} + \frac{1}{2} m\widehat{AB}$</td>
<td>6. Substitution</td>
</tr>
<tr>
<td>7. $m\angle ACB = \frac{1}{2} (m\widehat{AB} + m\widehat{DE})$</td>
<td>7. Distributive Property</td>
</tr>
</tbody>
</table>
26. Given: Secant $\overline{QT}$ and tangent $\overline{SR}$ intersect at point $S$.

Prove: $m \angle QSR = \frac{1}{2}(m\overline{QR} - m\overline{RT})$
27. Given: Tangents $\overline{VY}$ and $\overline{XY}$ intersect at point $Y$.
Prove: $m\angle Y = \frac{1}{2}(m\overarc{VW} - m\overarc{VX})$
28. Given: Chords $FI$ and $GH$ intersect at point $J$.
Prove: $m\angle FJH = \frac{1}{2}(m\angle FH + m\angle GI)$
29. Given: Secant $\overline{JL}$ and tangent $\overline{NL}$ intersect at point $L$.

Prove: $m\angle L = \frac{1}{2}(m\overarc{JM} - m\overarc{KM})$
30. Given: Tangents \( \overrightarrow{SX} \) and \( \overrightarrow{UX} \) intersect at point \( X \).

Prove: \( m \angle X = \frac{1}{2}(m\overline{TW} - m\overline{VW}) \)
Use the diagram shown to determine the measure of each angle or arc.

31. Determine $m\overarc{FI}$.
   $\angle K = 20^\circ$
   $m\overarc{GJ} = 80^\circ$
   $m\overarc{FI} = 120^\circ$

32. Determine $m\angle KLJ$.
   $\overarc{KM} = 120^\circ$
   $m\overarc{JN} = 100^\circ$

33. Determine $m\angle X$.
   $\overarc{VW} = 50^\circ$
   $m\overarc{TU} = 85^\circ$

34. Determine $m\angle WYX$.
   $\overarc{WUY} = 300^\circ$
35. Determine $m\overset{\frown}{RS}$.

$m\overset{\frown}{UV} = 30^\circ$

$m\angle RTS = 80^\circ$

36. Determine $m\angle D$.

$m\overset{\frown}{ZXC} = 150^\circ$

$m\overset{\frown}{CB} = 30^\circ$
Skills Practice

Name ____________________________ Date ________________

Color Theory
Chords

Vocabulary

Match each definition with its corresponding term.

1. Diameter-Chord Theorem
   a. If two chords of the same circle or congruent circles are congruent, then their corresponding arcs are congruent.

2. Equidistant Chord Theorem
   b. The segments formed on a chord when two chords of a circle intersect.

3. Equidistant Chord Converse Theorem
   c. If two chords of the same circle or congruent circles are congruent, then they are equidistant from the center of the circle.

4. Congruent Chord-Congruent Arc Theorem
   d. If two arcs of the same circle or congruent circles are congruent, then their corresponding chords are congruent.

5. Congruent Chord-Congruent Arc Converse Theorem
   e. If two chords of the same circle or congruent circles are equidistant from the center of the circle, then the chords are congruent.

6. segments of a chord
   f. If two chords of a circle intersect, then the product of the lengths of the segments of one chord is equal to the product of the lengths of the segments in the second chord.

7. Segment-Chord Theorem
   g. If a diameter of a circle is perpendicular to a chord, then the diameter bisects the chord and bisects the arc determined by the chord.
Problem Set

Use the given information to answer each question. Explain your answer.

1. If diameter $BD$ bisects $AC$, what is the angle of intersection?

The angle of intersection is $90^\circ$ because diameters that bisect chords are perpendicular bisectors.

2. If diameter $FH$ intersects $EG$ at a right angle, how does the length of $EI$ compare to the length of $IG$?

3. How does the measure of $KL$ and $LM$ compare?
4. If $\overline{KP} \cong \overline{LN}$, how does the length of $\overline{QO}$ compare to the length of $\overline{RO}$?

5. If $\overline{YO} \cong \overline{ZO}$, what is the relationship between $\overline{TU}$ and $\overline{XV}$?

6. If $\overline{GO} \cong \overline{HO}$ and diameter $\overline{EJ}$ is perpendicular to both, what is the relationship between $\overline{GF}$ and $\overline{HK}$?
Determine each measurement.

7. If $BD$ is a diameter, what is the length of $EC$?

$EC = EA = 5$ cm

8. If the length of $AB$ is 13 millimeters, what is the length of $CD$?

9. If the length of $AB$ is 24 centimeters, what is the length of $CD$?

10. If the length of $BF$ is 32 inches, what is the length of $CH$?
11. If the measure of $\angle AOB = 155^\circ$, what is the measure of $\angle DOC$?

![Diagram of circle with points A, B, O, D, C showing angles AOB and DOC]

12. If segment $\overline{AC}$ is a diameter, what is the measure of $\angle AED$?

![Diagram of circle with points A, C, O, B, E showing angles AED]

Compare each measurement. Explain your answer.

13. If $\overparen{DE} \cong \overparen{FG}$, how does the measure of $\overparen{DE}$ and $\overparen{FG}$ compare?

![Diagram of circle with points D, O, E, F, G showing chords DE and FG]

The measure of $\overparen{DE}$ is equal to the measure of $\overparen{FG}$ because congruent segments on the same circle create congruent chords.
14. If $\overline{KM} \cong \overline{JL}$, how does the measure of $\overline{JL}$ and $\overline{KM}$ compare?

15. If $\overline{QR} \cong \overline{PS}$, how does the measure of $\overline{QPR}$ and $\overline{PRS}$ compare?

16. If $\overline{EDG} \cong \overline{DEH}$, how does the measure of $\overline{EG}$ and $\overline{DH}$ compare?
17. If $\angle AOB \cong \angle DOC$, what is the relationship between $\overline{AB}$ and $\overline{DC}$?

18. If $\overline{EOH} \cong \overline{GOF}$, what is the relationship between $\overline{EH}$ and $\overline{FG}$?

Determine each measurement.

19. If $\overline{AB}$ is congruent to $\overline{CD}$, what is the length of $\overline{CD}$?

$\overline{CD} = \overline{AB} = 17 \text{ cm}$
20. If $\overline{EF}$ is congruent to $\overline{GH}$, what is the length of $\overline{EF}$?

![Diagram of a circle with chords $EF$ and $GH$ intersecting at a point inside the circle.]

21. If $\overline{XU}$ $\cong \overline{YV}$, what is the measure of $\overline{XU}$?

![Diagram of a circle with chords $XU$ and $YV$ intersecting at a point inside the circle.]

22. If $\overline{AB}$ $\cong \overline{CD}$, what is the measure of $\overline{AB}$?

![Diagram of a circle with chords $AB$ and $CD$ intersecting at a point inside the circle.]

23. If the length of $\overline{AB}$ is 24 centimeters, what is the length of $\overline{CD}$?

![Diagram of a circle with chords $AB$ and $CD$ intersecting at a point inside the circle.]
24. If the length of $EF$ is 14 millimeters, what is the length of $HG$?

Use the given circles to answer each question.

25. Name the segments of chord $KL$.

26. Name the segments of chord $MJ$.

27. Name the segments of chord $RU$.

28. Name the segments of chord $SV$.

29. Name the segments of chord $AC$.

30. Name the segments of chord $DB$. 
Use each diagram and the Segment Chord Theorem to write an equation involving the segments of the chords.

31. $DG \cdot GJ = FG \cdot GH$

33. $TK \cdot KO = KV \cdot VB$

35. $EI \cdot IO = IY \cdot YU$

32. $LQ \cdot QR = LO \cdot RO$

34. $SH \cdot HG = SV \cdot VC$

36. $XL \cdot JX = JO \cdot JC$
Skills Practice

Solar Eclipses
Tangents and Secants

Vocabulary

Write the term from the box that best completes each statement.

<table>
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<th>Secant-Segment Theorem</th>
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<tr>
<td>secant segment</td>
<td>Secant-Tangent Theorem</td>
<td>Tangent-Segment Theorem</td>
</tr>
</tbody>
</table>

1. A(n) _______ is the segment that is formed from an exterior point of a circle to the point of tangency.

2. The ___________________________ states that if two tangent segments are drawn from the same point on the exterior of the circle, then the tangent segments are congruent.

3. When two secants intersect in the exterior of a circle, the segment that begins at the point of intersection, continues through the circle, and ends on the other side of the circle is called a(n) ___________________________.

4. When two secants intersect in the exterior of a circle, the segment that begins at the point of intersection and ends where the secant enters the circle is called a(n) ___________________________.

5. The ___________________________ states that if two secants intersect in the exterior of a circle, then the product of the lengths of the secant segment and its external secant segment is equal to the product of the lengths of the second secant segment and its external secant segment.

6. The ___________________________ states that if a tangent and a secant intersect in the exterior of a circle, then the product of the lengths of the secant segment and its external secant segment is equal to the square of the length of the tangent segment.
Problem Set

Calculate the measure of each angle. Explain your reasoning.

1. If $\overline{OA}$ is a radius, what is the measure of $\angle OAB$?

A tangent line and the radius that ends at the point of tangency are perpendicular, so $m\angle OAB = 90^\circ$.

2. If $\overline{OD}$ is a radius, what is the measure of $\angle ODC$?

3. If $\overline{YO}$ is a radius, what is the measure of $\angle YXO$?
4. If $RS$ is a tangent segment and $OS$ is a radius, what is the measure of $\angle ROS$?

5. If $UT$ is a tangent segment and $OU$ is a radius, what is the measure of $\angle TOU$?
6. If $\overline{VW}$ is a tangent segment and $\overline{OV}$ is a radius, what is the measure of $\angle VWO$?

![diagram]

Write a statement to show the congruent segments.

7. $\overline{AC}$ and $\overline{BC}$ are tangent to circle $O$.

8. $\overline{XZ}$ and $\overline{ZW}$ are tangent to circle $O$.

$\overline{AC} \cong \overline{CB}$

9. $\overline{RS}$ and $\overline{RT}$ are tangent to circle $O$.

10. $\overline{MP}$ and $\overline{NP}$ are tangent to circle $O$. 
11. $\overline{DE}$ and $\overline{FE}$ are tangent to circle $O$.

12. $\overline{GH}$ and $\overline{GI}$ are tangent to circle $O$.

Calculate the measure of each angle. Explain your reasoning.

13. If $\overline{EF}$ and $\overline{GF}$ are tangent segments, what is the measure of $\angle EGF$?

Because $\overline{EF}$ and $\overline{GF}$ are tangent segments drawn from point $F$, they must be congruent. This fact means that $\triangle EFG$ is an isosceles triangle.

$m \angle EGF = (180^\circ - 64^\circ) \div 2$

$= 116^\circ \div 2 = 58^\circ$

The measure of $\angle EGF$ is $58^\circ$. 
14. If $\overline{HI}$ and $\overline{JI}$ are tangent segments, what is the measure of $\angle HJI$?

15. If $\overline{KM}$ and $\overline{LM}$ are tangent segments, what is the measure of $\angle KML$?
16. If \( \overline{NP} \) and \( \overline{QP} \) are tangent segments, what is the measure of \( \angle NPQ \)?

17. If \( \overline{AF} \) and \( \overline{VF} \) are tangent segments, what is the measure of \( \angle AVF \)?
18. If $\overline{RT}$ and $\overline{MT}$ are tangent segments, what is the measure of $\angle RTM$?

![Diagram of a circle with tangent segments RT and MT, and an angle at M.]

19. Name two secant segments and two external secant segments for circle $O$.

Secant segments: $\overline{PT}$ and $\overline{QT}$
External secant segments: $\overline{RT}$ and $\overline{ST}$

20. 

21. 

22. 

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Use each diagram and the Secant Segment Theorem to write an equation involving the secant segments.

25. \( RV \cdot TV = SV \cdot UV \)
29. 

30. 

Name a tangent segment, a secant segment, and an external secant segment for circle O.

31. 

32. 

Tangent segment: $\overline{TU}$
Secant segment: $\overline{RT}$
External secant segment: $\overline{ST}$

33. 

34.
Use each diagram and the Secant Tangent Theorem to write an equation involving the secant and tangent segments.

37. \((EM)^2 = QM \cdot WM\)
41.

42.
Skills Practice

Replacement for a Carpenter's Square
Inscribed and Circumscribed Triangles and Quadrilaterals

Vocabulary

Answer each question.

1. How are inscribed polygons and circumscribed polygons different?

2. Describe how you can use the Inscribed Right Triangle-Diameter Theorem to show an inscribed triangle is a right triangle.

3. What does the Converse of the Inscribed Right Triangle-Diameter Theorem help to show in a circle?

4. What information about a quadrilateral inscribed in a circle does the Inscribed Quadrilateral-Opposite Angles Theorem give?
Problem Set

Draw a triangle inscribed in the circle through the three points. Then determine if the triangle is a right triangle.

1. No. The triangle is not a right triangle. None of the sides of the triangle is a diameter of the circle.
7. Draw a triangle inscribed in the circle through the given points. Then determine the measure of the indicated angle.

8. In $\triangle ABC$, $m\angle A = 55^\circ$. Determine $m\angle B$.

$$m\angle B = 180^\circ - 90^\circ - 55^\circ = 35^\circ$$

9. In $\triangle ABC$, $m\angle A = 55^\circ$. Determine $m\angle B$.

10. In $\triangle ABC$, $m\angle B = 38^\circ$. Determine $m\angle A$. 
11. In $\triangle ABC$, $m \angle C = 62^\circ$. Determine $m \angle A$.

12. In $\triangle ABC$, $m \angle B = 26^\circ$. Determine $m \angle C$.

13. In $\triangle ABC$, $m \angle C = 49^\circ$. Determine $m \angle A$.

14. In $\triangle ABC$, $m \angle B = 51^\circ$. Determine $m \angle A$. 
Draw a quadrilateral inscribed in the circle through the given four points. Then determine the measure of the indicated angle.

15. In quadrilateral $ABCD$, $m\angle B = 81^\circ$. Determine $m\angle D$.

\[
m\angle D = 180^\circ - 81^\circ = 99^\circ
\]

16. In quadrilateral $ABCD$, $m\angle C = 75^\circ$. Determine $m\angle A$.

17. In quadrilateral $ABCD$, $m\angle B = 112^\circ$. Determine $m\angle D$.  

18. In quadrilateral $ABCD$, $m \angle D = 93^\circ$. Determine $m \angle B$.

19. In quadrilateral $ABCD$, $m \angle A = 72^\circ$. Determine $m \angle C$.

20. In quadrilateral $ABCD$, $m \angle B = 101^\circ$. Determine $m \angle D$. 
Construct a circle inscribed in each polygon.

21.

22.
23.

24.

25.
Create a two-column proof to prove each statement.

26. Create a two-column proof to prove each statement.

27. Given: Inscribed $\triangle ABC$ in circle $O$, $m\overset{\frown}{AC} = 40^\circ$, and $m\overset{\frown}{BC} = 140^\circ$

Prove: $\overline{AB}$ is a diameter of circle $O$.

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<td>1. Given</td>
</tr>
<tr>
<td>2. $m\overset{\frown}{AC} + m\overset{\frown}{BC} + m\overset{\frown}{AB} = 360^\circ$</td>
<td>2. Arc Addition Postulate</td>
</tr>
<tr>
<td>3. $40^\circ + 140^\circ + m\overset{\frown}{AB} = 360^\circ$</td>
<td>3. Substitution</td>
</tr>
<tr>
<td>4. $m\overset{\frown}{AB} = 180^\circ$</td>
<td>4. Subtraction Property of Equality</td>
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<tr>
<td>5. $m\angle C = \frac{1}{2} m\overset{\frown}{AB}$</td>
<td>5. Definition of inscribed angle</td>
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<tr>
<td>6. $m\angle C = 90^\circ$</td>
<td>6. Substitution</td>
</tr>
<tr>
<td>7. $\triangle ABC$ is a right triangle with right angle $C$.</td>
<td>7. Definition of right triangle</td>
</tr>
<tr>
<td>8. $\overline{AB}$ is the diameter of circle $O$.</td>
<td>8. Converse of Inscribed Right Triangle-Diameter Theorem</td>
</tr>
</tbody>
</table>
28. Given: Inscribed $\triangle RST$ in circle $O$ with diameter $RS$, and $m\overline{RT} = 60^\circ$

Prove: $m\overline{ST} = 120^\circ$
29. Given: Inscribed quadrilateral \(ABCD\) in circle \(O\), \(m\overline{AB} = 50^\circ\), and \(m\overline{BC} = 90^\circ\)

Prove: \(m\angle B = 110^\circ\)
30. Given: Inscribed quadrilateral $MNPQ$ in circle $O$, $m\overarc{MQ} = 75^\circ$, and $m\angle NMQ = 120^\circ$
Prove: $m\overarc{MN} = 45^\circ$
31. Given: \( m\overline{AB} = 50^\circ \), \( m\overline{BC} = 90^\circ \), and \( m\overline{CD} = 90^\circ \)

Prove: \( m\angle BCD = 90^\circ \)
32. Given: \( \angle BAD \) and \( \angle ADC \) are supplementary angles

Prove: \( m\angle BAD = m\angle ABC \)
Skills Practice

Gears
Arc Length

Vocabulary
Define the key term in your own words.

1. arc length

Problem Set
Calculate the ratio of the length of each arc to the circle's circumference.

1. The measure of $\overline{AB}$ is 40°.
   \[
   \frac{40^\circ}{360^\circ} = \frac{1}{9}
   \]
   The arc is $\frac{1}{9}$ of the circle's circumference.

2. The measure of $\overline{CD}$ is 90°.

3. The measure of $\overline{EF}$ is 120°.

4. The measure of $\overline{GH}$ is 150°.

5. The measure of $\overline{IJ}$ is 105°.

6. The measure of $\overline{KL}$ is 75°.
Write an expression that you can use to calculate the length of \( \widehat{RS} \). You do not need to simplify the expression.

7. \[ \frac{80}{360} \cdot 2\pi(15) \]

8. \[ \frac{110}{360} \cdot 2\pi(15) \]

9. \[ \frac{90}{360} \cdot 2\pi(17) \]

10. \[ \frac{160}{360} \cdot 2\pi(28) \]

11. \[ \frac{180}{360} \cdot 2\pi(63) \]

12. \[ \frac{150}{360} \cdot 2\pi(33) \]

Calculate each arc length. Write your answer in terms of \( \pi \).

13. If the measure of \( \widehat{AB} \) is 45° and the radius is 12 meters, what is the arc length of \( \widehat{AB} \)?

\[ C = 2\pi(12) = 24\pi \]

Fraction of \( C \): \[ \frac{45}{360} = \frac{1}{8} \]

Arc length of \( \widehat{AB} \): \[ \frac{1}{8} (24\pi) = 3\pi \]

The arc length of \( \widehat{AB} \) is 3\( \pi \) meters.
14. If the measure of $\widehat{CD}$ is 120º and the radius is 15 centimeters, what is the arc length of $\widehat{CD}$?

15. If the measure of $\widehat{EF}$ is 60º and the radius is 8 inches, what is the arc length of $\widehat{EF}$?

16. If the measure of $\widehat{GH}$ is 30º and the radius is 6 meters, what is the arc length of $\widehat{GH}$?

17. If the measure of $\widehat{IJ}$ is 80º and the diameter is 10 centimeters, what is the arc length of $\widehat{IJ}$?

18. If the measure of $\widehat{KL}$ is 15º and the diameter is 18 feet, what is the arc length of $\widehat{KL}$?
19. If the measure of $\hat{MN}$ is 75º and the diameter is 20 millimeters, what is the arc length of $\hat{MN}$?

20. If the measure of $\hat{OP}$ is 165º and the diameter is 21 centimeters, what is the arc length of $\hat{OP}$?

**11**

Calculate each arc length. Write your answer in terms of $\pi$.

21. If the measure of $\hat{AB}$ is 135º, what is the arc length of $\hat{AB}$?

\[
C = 2\pi(16) = 32\pi
\]

Fraction of $C$: \[\frac{135^\circ}{360^\circ} = \frac{3}{8}\]

Arc length of $\hat{AB}$: \[\frac{3}{8}(32\pi) = \frac{96}{8}\pi = 12\pi\]

The arc length of $\hat{AB}$ is $12\pi$ cm.
22. If the measure of $\overarc{CD}$ is $45^\circ$, what is the arc length of $\overarc{CD}$?

23. If the measure of $\overarc{EF}$ is $90^\circ$, what is the arc length of $\overarc{EF}$?
24. If the measure of $\widehat{GH}$ is $120^\circ$, what is the arc length of $\widehat{GH}$?

25. If the length of the radius is 4 centimeters, what is the arc length of $\widehat{IJ}$?
26. If the length of the radius is 7 centimeters, what is the arc length of $\overline{KL}$?

27. If the length of the radius is 11 centimeters, what is the arc length of $\overline{MN}$?
28. If the length of the radius is 17 centimeters, what is the arc length of $\overparen{OP}$?

![Diagram](image)

Use the given information to answer each question. Where necessary, use 3.14 to approximate $\pi$.

29. Determine the perimeter of the shaded region.

![Diagram](image)

Arc length: $\frac{60}{360} \times 2(3.14)(30) = 31.4$ mm

Perimeter of shaded region: $31.4 + 30 + 30 = 91.4$ mm
30. Determine the perimeter of the figure below.

![Diagram of a figure with dimensions 15 cm, 30 cm, and 180°.]

31. A semicircular cut was taken from the rectangle shown. Determine the perimeter of the shaded region.

![Diagram of a rectangle with dimensions 80 ft and 60 ft, with a semicircle cut out.]

32. A circle has a circumference of 81.2 inches. What is the radius of the circle?
33. Bella used a tape measure and found the circumference of a flagpole to be 6.2 inches. What is the radius of the flagpole?

34. Carla used a string and a tape measure and found the circumference of a circular cup to be 12.56 inches. What is the radius of the cup?
Skills Practice

Name ___________________________ Date ______________

Playing Darts
Sectors and Segments of a Circles

Vocabulary

Draw an example of each term.

1. concentric circles
2. sector of a circle
3. segment of a circle

Problem Set

Calculate the area of each circle. Write your answer in terms of $\pi$.

1. What is the area of a circle whose radius is 5 centimeters?
   \[ A = \pi(5^2) = 25\pi \]
   The area of the circle is $25\pi$ cm$^2$.

2. What is the area of a circle whose radius is 8 millimeters?
3. What is the area of a circle whose radius is 12 feet?

4. What is the area of a circle whose radius is 18 centimeters?

5. What is the area of a circle whose diameter is 22 inches?

6. What is the area of a circle whose diameter is 28 meters?

7. What is the area of a circle whose diameter is 15 inches?

8. What is the area of a circle whose diameter is 31 yards?
Calculate the area of each sector. Write your answer in terms of $\pi$.

9. If the radius of the circle is 9 centimeters, what is the area of sector $AOB$?

![Diagram of a circle with a 120° angle at O]

Total area of the circle = $\pi(9^2) = 81\pi$ cm$^2$

Sector $AOB$'s fraction of the circle = $\frac{120^\circ}{360^\circ} = \frac{1}{3}$

Area of sector $AOB = \frac{1}{3}(81\pi) = 27\pi$

The area of sector $AOB$ is $27\pi$ cm$^2$.

10. If the radius of the circle is 16 meters, what is the area of sector $COD$?

![Diagram of a circle with a 45° angle at O]
11. If the radius of the circle is 15 feet, what is the area of sector $EOF$?

![Diagram of a circle with angles and segments]

12. If the radius of the circle is 10 inches, what is the area of sector $GOH$?

![Diagram of a circle with angles and segments]
13. If the radius of the circle is 32 centimeters, what is the area of sector $IOJ$?

14. If the radius of the circle is 20 millimeters, what is the area of sector $KOL$?
15. If the radius of the circle is 24 centimeters, what is the area of sector $MON$?

16. If the radius of the circle is 21 meters, what is the area of sector $POQ$?
Calculate the area of each segment. Round your answer to the nearest tenth, if necessary. Use 3.14 to estimate \( \pi \).

17. If the radius of the circle is 6 centimeters, what is the area of the shaded segment?

Total area of the circle = \( \pi (6^2) = 36\pi \) cm\(^2\)

Sector AOB’s fraction of the circle = \( \frac{90^\circ}{360^\circ} = \frac{1}{4} \)

Area of sector AOB = \( \frac{1}{4} (36\pi) = 9\pi \) cm\(^2\)

Area of \( \triangle AOB = \frac{1}{2} (6 \cdot 6) = 18 \) cm\(^2\)

Area of the segment: \( 9\pi - 18 \approx 28.3 - 18 = 10.3 \)

The area of the shaded segment is approximately 10.3 cm\(^2\).

18. If the radius of the circle is 14 inches, what is the area of the shaded segment?
19. If the radius of the circle is 17 feet, what is the area of the shaded segment?

![Diagram of a circle with a shaded segment and a 90° angle at point O.]

20. If the radius of the circle is 22 centimeters, what is the area of the shaded segment?

![Diagram of a circle with a shaded segment and a 90° angle at point O.]
21. If the radius of the circle is 25 meters, what is the area of the shaded segment?

22. If the radius of the circle is 30 centimeters, what is the area of the shaded segment?
In circle $O$ below, $m\widehat{AB} = 90^\circ$. Use the given information to determine the length of the radius of circle $O$.

23. If the area of the segment is $16\pi - 32$ square feet, what is the length of the radius of circle $O$?

\[
16\pi = \frac{90^\circ}{360^\circ}(\pi r^2)
\]

\[
16\pi = \frac{1}{4}(\pi r^2)
\]

\[
64 = r^2
\]

\[
8 = r
\]

The length of the radius is 8 feet.

24. If the area of the segment is $25\pi - 50$ square inches, what is the length of the radius of circle $O$?

25. If the area of the segment is $\pi - 2$ square meters, what is the length of the radius of circle $O$?
26. If the area of the segment is $56.25\pi - 112.5$ square yards, what is the length of the radius of circle $O$?

27. If the area of the segment is $121\pi - 242$ square feet, what is the length of the radius of circle $O$?

28. If the area of the segment is $90.25\pi - 180.5$ square millimeters, what is the length of the radius of circle $O$?
Problem Set

Use the given information to show that each statement is true. Justify your answers by using theorems and by using algebra.

1. The center of circle $O$ is at the origin. The coordinates of the given points are $A(-4, 0)$, $B(4, 0)$, and $C(0, 4)$. Show that $\triangle ABC$ is a right triangle.

$\triangle ABC$ is an inscribed triangle in circle $O$ with the hypotenuse as the diameter of the circle, therefore the triangle is a right triangle by the Diameter-Right Triangle Converse Theorem.

$AB = \sqrt{(-4 - 4)^2 + (0 - 0)^2}$
$= \sqrt{(-8)^2} = \sqrt{64} = 8$

$AC = \sqrt{(-4 - 0)^2 + (0 - 4)^2}$
$= \sqrt{(-4)^2 + (-4)^2}$
$= \sqrt{32} = 4\sqrt{2}$

$BC = \sqrt{(4 - 0)^2 + (0 - 4)^2}$
$= \sqrt{4^2 + (-4)^2}$
$= \sqrt{32} = 4\sqrt{2}$

$(4\sqrt{2})^2 + (4\sqrt{2})^2 = 8^2$
$32 + 32 = 64$
2. The center of circle $O$ is at the origin. Lines $\overline{AZ}$ and $\overline{AT}$ are tangent to circle $O$. The coordinates of the given points are $A(-10, 30)$, $T(8, 6)$, and $Z(-10, 0)$. Show that the lengths of $\overline{AT}$ and $\overline{AZ}$ are equal.

3. The center of circle $C$ is at the origin. Line $\overline{AB}$ is tangent to circle $C$ at $(3, 4)$. Show that the tangent line is perpendicular to $\overline{CA}$.
4. The center of circle $O$ is at the origin. The coordinates of the given points are $A(3, 4)$, $B(-4, -3)$, $D(0, -5)$, $E(-3, 4)$, and $F(-1.5, -0.5)$. Show that $EF \cdot FD = AF \cdot FB$. 

![Diagram with labeled points A, B, C, D, E, and F.]
5. The center of circle $O$ is at the origin. The coordinates of the given points are $A(-5, 5)$, $B(-4, 3)$, $C(0, -5)$, and $D(0, 5)$. Show that $AD^2 = AB \cdot AC$. 
6. The center of circle $O$ is at the origin. Chord $BD$ is perpendicular to radius $OA$ at point $C$. The coordinates of the given points are $A(-5, 0)$, $B(-4, 3)$, $C(-4, 0)$, and $D(-4, -3)$. Show that $OA$ bisects $BD$. 
Classify the polygon formed by connecting the midpoints of the sides of each quadrilateral. Show all your work.

7. The rectangle shown has vertices \(A(0, 0), B(x, 0), C(x, y),\) and \(D(0, y)\).

Midpoint \(\overline{AB} : U\left(\frac{x}{2}, 0\right)\)
Midpoint \(\overline{BC} : S\left(x, \frac{y}{2}\right)\)
Midpoint \(\overline{CD} : T\left(\frac{x}{2}, y\right)\)
Midpoint \(\overline{DA} : R\left(0, \frac{y}{2}\right)\)

Slope \(\overline{RT} = \frac{y - 2}{x - 0} = \frac{2}{x} = \frac{y}{x}\)
Slope \(\overline{TS} = \frac{y - 2}{x - \frac{x}{2}} = \frac{2}{x} = \frac{y}{x}\)
Slope \(\overline{US} = \frac{0 - \frac{y}{2}}{2 - x} = -\frac{y}{2} = \frac{y}{-x}\)
Slope \(\overline{RU} = \frac{0 - \frac{y}{2}}{x - 2} = -\frac{y}{2} = \frac{y}{-x}\)

Sides \(\overline{RT}\) and \(\overline{US}\) are parallel since they have the same slope of \(\frac{y}{x}\).
Sides \(\overline{TS}\) and \(\overline{RU}\) are parallel since they have the same slope of \(-\frac{y}{x}\).

There are no perpendicular sides. The slopes are not negative reciprocals.

\[
\overline{RT} = \sqrt{\left(0 - \frac{x}{2}\right)^2 + \left(\frac{y}{2} - y\right)^2} \\
= \sqrt{-\frac{x^2}{4} + \frac{y^2}{4}} \\
= \sqrt{\frac{x^2}{4} + \frac{y^2}{4}}
\]

\[
\overline{US} = \sqrt{\left(x - \frac{x}{2}\right)^2 + \left(y - 0\right)^2} \\
= \sqrt{\frac{x^2}{4} + \frac{y^2}{4}}
\]

\[
\overline{TS} = \sqrt{\left(\frac{x}{2} - x\right)^2 + \left(y - y\right)^2} \\
= \sqrt{-\frac{x^2}{4} + \frac{y^2}{4}} \\
= \sqrt{\frac{x^2}{4} + \frac{y^2}{4}}
\]

\[
\overline{RU} = \sqrt{\left(0 - x\right)^2 + \left(\frac{y}{2} - 0\right)^2} \\
= \sqrt{-\frac{x^2}{4} + \frac{y^2}{4}} \\
= \sqrt{\frac{x^2}{4} + \frac{y^2}{4}}
\]

All four sides of the new quadrilateral are congruent.

Opposite sides are parallel and all sides are congruent, so the quadrilateral formed by connecting the midpoints of the rectangle is a rhombus.
8. The isosceles trapezoid shown has vertices $A(0, 0)$, $B(6, 0)$, $C(4, 3)$, and $D(2, 3)$. 
9. The parallelogram shown has vertices $A(4, 5)$, $B(7, 5)$, $C(3, 0)$, and $D(0, 0)$. 
10. The rhombus shown has vertices $A(4, 10)$, $B(8, 5)$, $C(4, 0)$, and $D(0, 5)$. 
11. The square shown has vertices $A(0, 0)$, $B(4, 0)$, $C(4, 4)$, and $D(0, 4)$. 
12. The kite shown has vertices $A(0, 2)$, $B(2, 0)$, $C(0, -3)$, and $D(-2, 0)$. 